

Assignment for Incoming AP Calculus Students

NO CALCULATOR

Please show all the work in the space provided.

I. ALGEBRAIC PROPERTIES

Which of the following statements are TRUE for all real values of x ? For those that are FALSE, illustrate with a specific example (a counter example) to show that it is false.

1) $x^3 + 1 > x^3$

3) $x^3 + x > x^3$

5) $x^2 \geq 0$

2) $-x \leq 0$

4) $\frac{1}{x} \leq x$

6) $x \leq |x|$

II. FACTORING

1) $t^3 - 8$

3) $6x^2 + 7x - 3$

5) $\frac{1}{2}t^2 + t - 12$

7) $x^5 + 11x^3 - 80x$

8. $(x-3)^2(2x+1)^3 + (x-3)^3(2x+1)^2$

9. $2x^2 + 50y^2 - 20xy$

III. Solve each equation by factoring or using the quadratic formula.

1. $7x^2 - 3x = 0$ 2. $4x(x-2) - 5x(x-1) = 2$ 3. $x^2 + 6x + 4 = 0$

4. $x^4 - 9x^2 + 8 = 0$ 5. $x - 10\sqrt{x} + 9 = 0$ 6. $\frac{1}{x^2} - \frac{1}{x} = 6$

IV. SOLVING EQUATIONS

1) Solve for x in terms of y : $\sqrt{x^2 - xy} = 1 + x$

2) Solve for x in terms of y : $y^2 = \frac{1-x}{2+x}$

3) Solve for y in terms of x : $x^2 = \ln y - \ln 3$

4) Solve for a in terms of x and y : $2x + xa - y = 1 - 2ay$

V.EXPONENTIAL AND LOGARITHMIC FUNCTIONS

Simplify each expression:

1) $e^{\ln e^x} =$

5) $e^{\ln \frac{1}{x}} =$

9) $\ln \frac{1}{e^3} =$

3) $e^{1-\ln x} =$

7) $\ln \frac{1}{e} =$

11) $e^{x+\ln x} =$

4) $e^{3+\ln 4} =$

8) $e^{2\ln x} =$

12) $e^{(\ln x - 2\ln y)} =$

Solve for x algebraically:

1) $\ln(x+1) + \ln x = \ln 6$ 4) $\ln(x-1) + \ln 2 = 0$ 7) $2 \ln 3x = 4$

2) $\ln(x) - \ln(x-2) = 1$ 5) $\ln x - \ln(\frac{1}{x}) = 2$ 8) $3e^{x+2} = 12$

3) $\ln x^2 - 1 = 3$ 6) $10 = 5e^{5x}$ 9) $e^{2x} - 3e^x + 2 = 0$

VI. AREA OF GEOMETRIC FIGURES

- 1) Find the area of an equilateral triangle whose perimeter is 15.
- 2) Find the area of a square whose diagonal is 3.
- 3) Find the area of a trapezoid whose bases are 2 and 3, and whose height is 4.
- 4) Find the surface area of a closed right circular cylinder with radius 2 and height 5.
- 5) Express, in terms of x , the area of a semi-circle whose diameter is x .
- 6) Express, in terms of x , the area of a circle whose circumference is x .
- 7) Express, in terms of x , the area of a square circumscribed around a circle whose area is x .

VII. TRIGONOMETRY

1) In which quadrant does angle x lie if $\sin x < 0$ and $\tan x > 0$?

2) Find the exact numerical value:

a) $\cos \frac{\pi}{3} - \sin \frac{3\pi}{2}$ d) $\tan \frac{\pi}{4} - \sin \frac{\pi}{2}$ g) $\sin \frac{3\pi}{2} - \cos 3\pi$ j) $\tan \frac{5\pi}{4} + \cos 0$

b) $\sin 2\pi + \cos \pi$ e) $-\cos(-\pi) + \sin\left(-\frac{3\pi}{2}\right)$ h) $\tan \frac{\pi}{3} + \tan \frac{\pi}{6}$ k) $\cos\left(-\frac{\pi}{4}\right) + \sin \frac{2\pi}{3}$

c) $\sin \frac{\pi}{2} + \cos \frac{\pi}{4}$ f) $\csc \frac{\pi}{3} + \sec \frac{\pi}{6}$ i) $\sin \frac{3\pi}{4} - \cos \frac{5\pi}{4}$ l) $(\cot \frac{\pi}{6})(\cos \frac{7\pi}{4})$

2) If $f(x) = \sin x + \cos 2x$, find the exact numerical value of $f\left(\frac{\pi}{6}\right)$.

4) Express each in simplest form:

a) $\tan \theta \csc \theta$ d) $\frac{\sin 2A}{2 \cos^2 A}$ g) $\sin \theta + \frac{\cos^2 \theta}{\sin \theta}$

b) $\cot \theta \csc \theta \sec \theta$ e) $\frac{\cos x}{\sec x} + \frac{\sin x}{\csc x}$ h) $\sin(A-B) + \sin(A+B)$

c) $\csc \theta - \cos \theta \cot \theta$ f) $\frac{\sec x + \csc x}{1 + \tan x}$ i) $\left(\frac{1}{\cos \theta} - \frac{\sin \theta}{\tan \theta} \right) (\cos \theta)$

5) Evaluate each expression:

a) $\tan(\cos^{-1} \frac{3}{5})$ c) $\sin(\tan^{-1} \frac{2}{3})$

b) $\cos(\tan^{-1} \frac{5}{12})$ d) $\sin(\cos^{-1} \frac{1}{5})$

6) Express in degrees an angle of $\frac{3\pi}{5}$ radians.

7) Express in radians an angle of 210° .

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8) Find the smallest positive value of x for which $\cot x = 3 \tan x$.

9) If θ is an angle in the second quadrant, and if $\cos \theta = -\frac{4}{5}$, find the value of $\sin \theta$.

10) Find the smallest positive value of A for which $\cos A = \frac{1}{\csc A}$.

11) If θ is a positive acute angle, express $\cos \theta$ in terms of $\tan \theta$.

12) Find all values of x on the interval $[0, 2\pi]$ for which $\sin^2 x = 2 \sin x$.

13) Find all values of x on the interval $[0, 2\pi]$ for which $\sqrt{8 - 2 \sin \theta} = 3$.

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14) If x is a positive acute angle, and $\sin x = \frac{2}{\sqrt{13}}$, find $\sin 2x$.

15) Find the positive acute value of θ for which $\sec^2 \theta - 4 = 0$.

16) If $\cos x = a$, express $\cos 2x$ in terms of a .

17) Find all values of x on the interval $[0, \pi]$ for which $1 - \cos 2x = 2 \sin^2 x$?

18) Find all values of x on the interval $[0, 2\pi]$ for which $\tan^2 x = 1$?

VIII. FUNCTIONS

1) If $f(x) = x^{\frac{4}{3}} + x^{\frac{2}{3}} + x^0$, what is the value of $f(-8)$?

2) If $f(x) = \frac{3x+2}{x+1}$, find $f^{-1}(x)$.

3) If $f(x) = 2^x$, find $f^{-1}(x)$.

4) If $f(x) = e^x$, find $f^{-1}(x)$.

6) Identify each of the following function as Even, Odd, or Neither:

a) $y = x^2 - 1$ c) $y = \cos x$ e) $y = x^3$ g) $y = \ln x$

b) $y = \sin x$ d) $y = x^3 + 3$ f) $y = x + \sin x$ h) $y = e^{-x}$

7) Write an equation for the line whose slope is $\frac{2}{3}$, and which passes through the point $(4, -1)$.

8) Write an equation for the line which passes through the points $(2, -3)$ and $(-4, 5)$.

- 9) Write an equation for the line which passes through the point $(-1, 2)$ and is parallel to the line whose equation is $3y - 2x + 6 = 0$.
- 10) Write an equation for the line which passes through the point $(6, -3)$ and is perpendicular to the line whose equation is $x - 2y = 1$.

IX. DOMAINS *(use interval notation where appropriate)*

1. $y = \log(2x - 12)$ 2. $y = \frac{x^2 - 4}{2x + 4}$ 3. $y = \frac{x^2 - 5x - 6}{x^2 - 3x - 18}$ 4. $y = \frac{2^{2-x}}{x}$

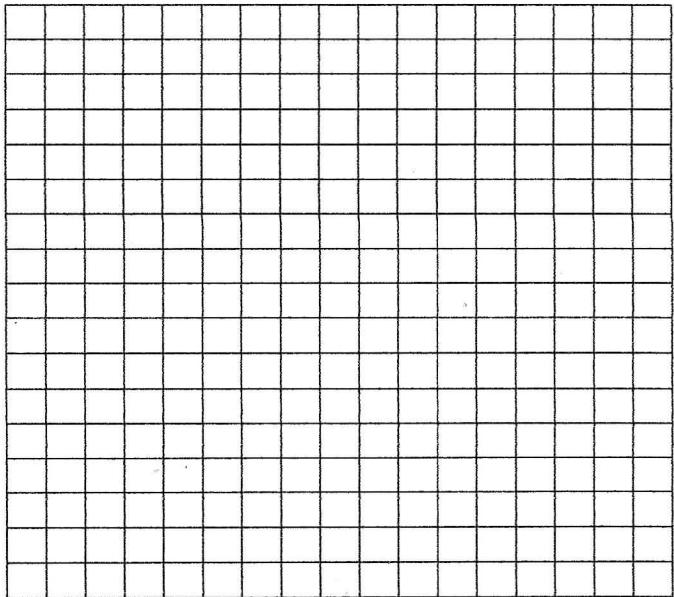
5. $y = \sqrt{x-3} - \sqrt{x+3}$ 6. $y = \frac{\sqrt{2x-9}}{2x+9}$ 7. $y = \sqrt{\tan x}$

9. $y = \sqrt{x^2 - 5x - 14}$ 10. $y = \frac{3x-2}{4x+1}$ 12. $y = \frac{x}{\cos x}$

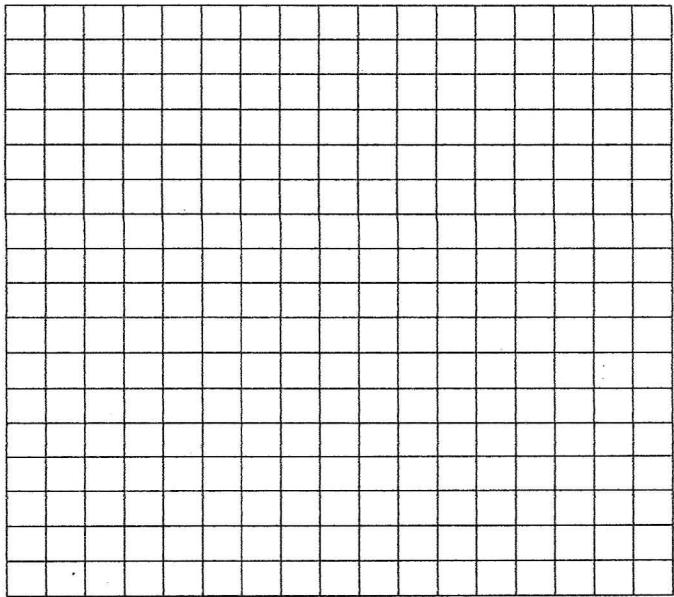
X.GRAPHS

Sketch each of the following graphs (Remember--no calculator)

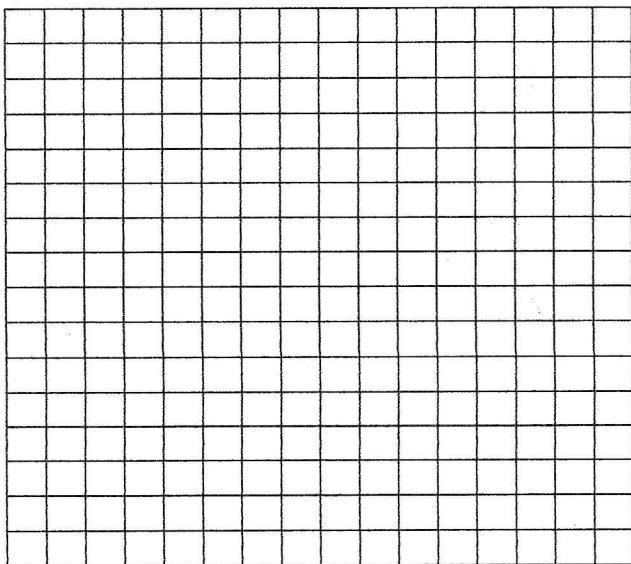
1) $y = x$



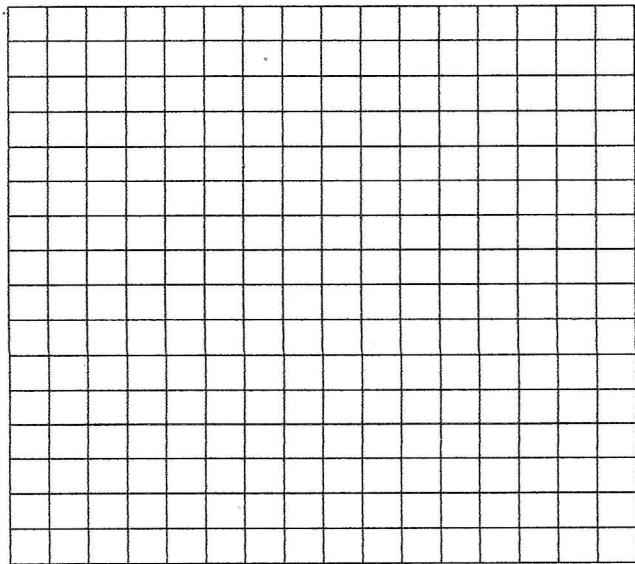
2) $y = x^2$



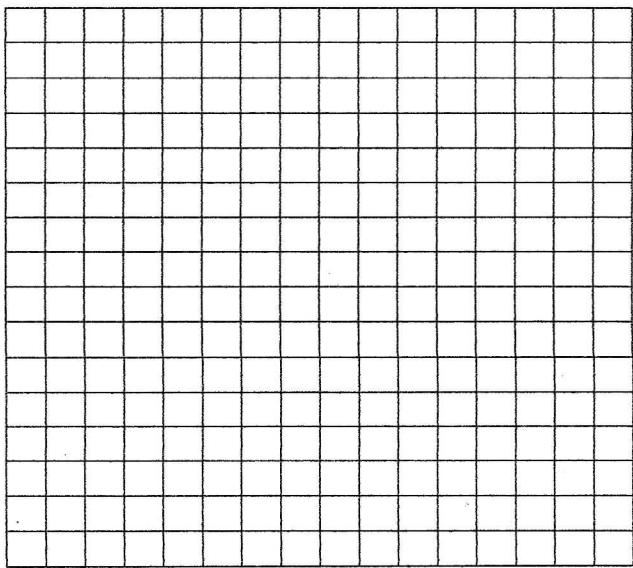
3) $y = x^3$



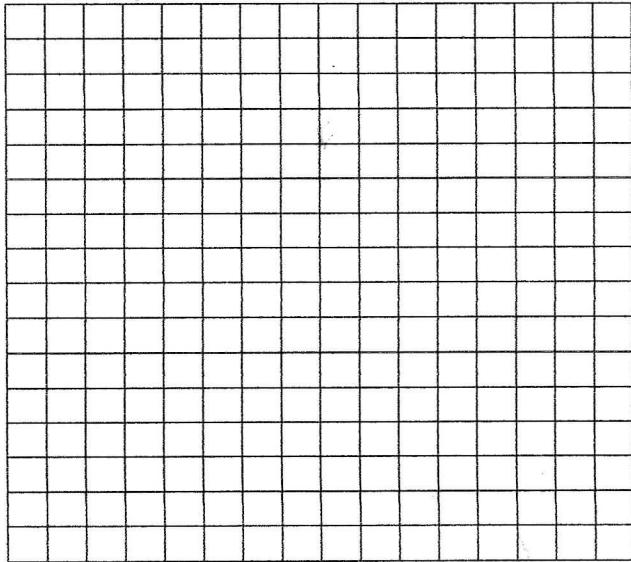
4) $y = \sqrt{x}$



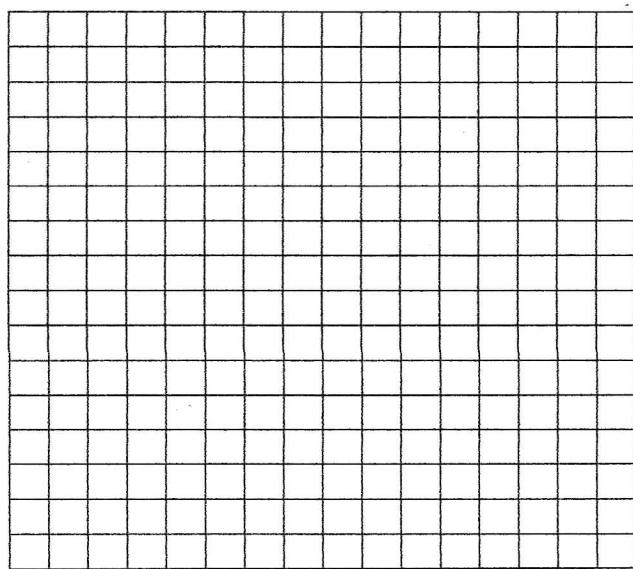
5) $y = \sqrt{1 - x^2}$



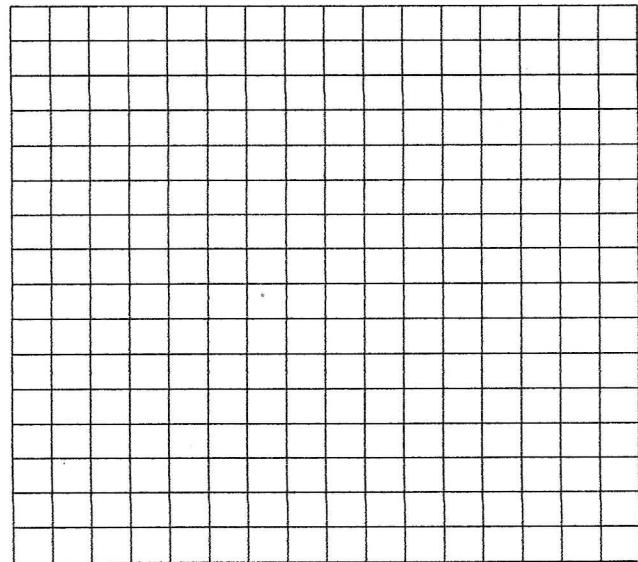
7) $y = \frac{1}{x}$



6) $y = |x|$

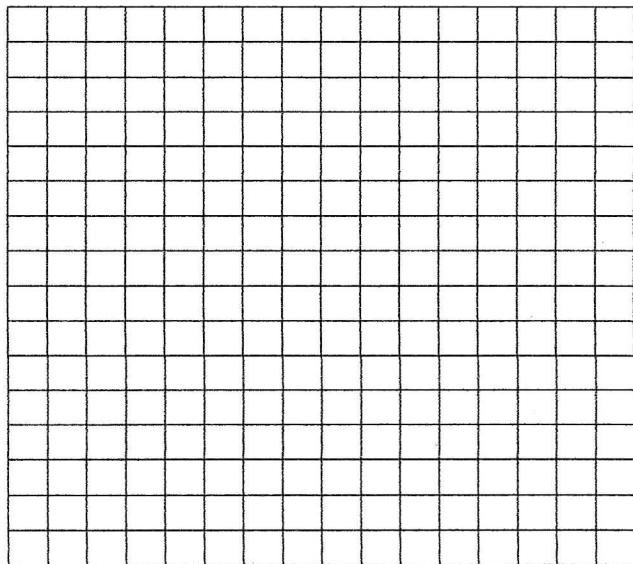
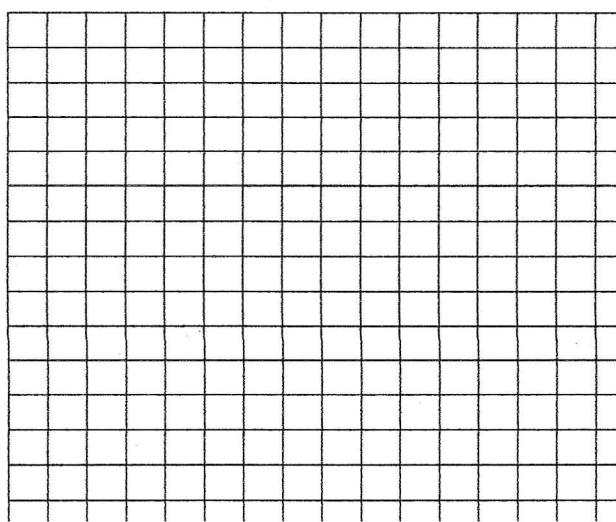
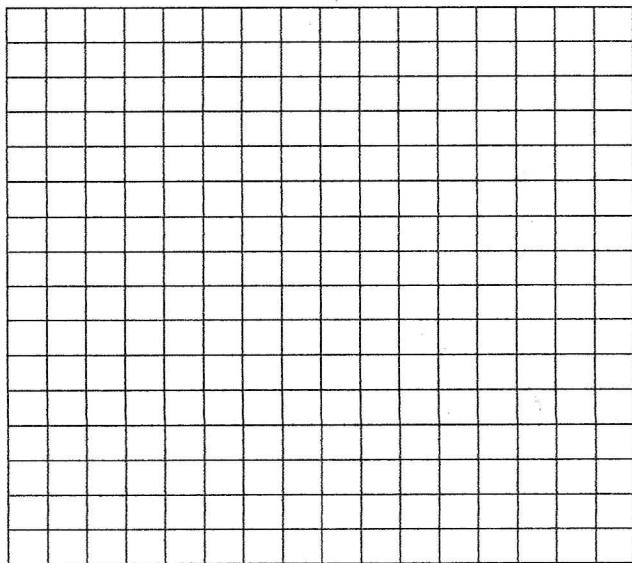
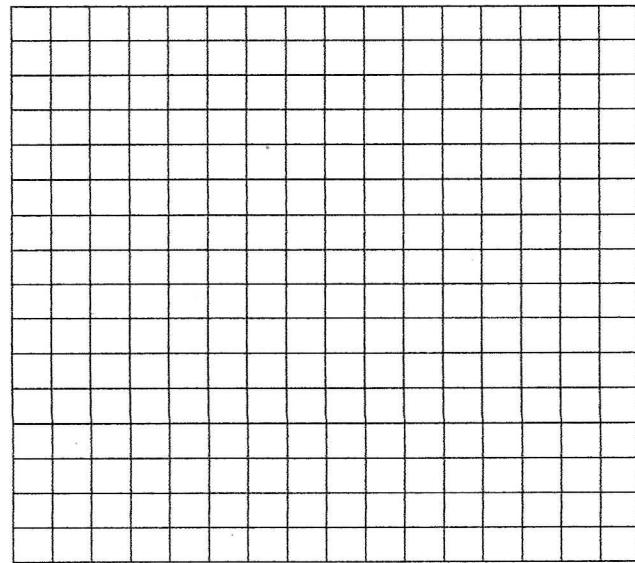


8) $y = e^x$

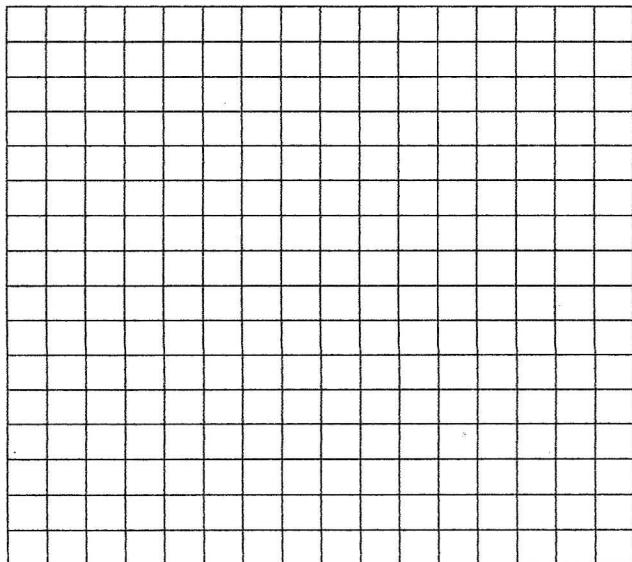


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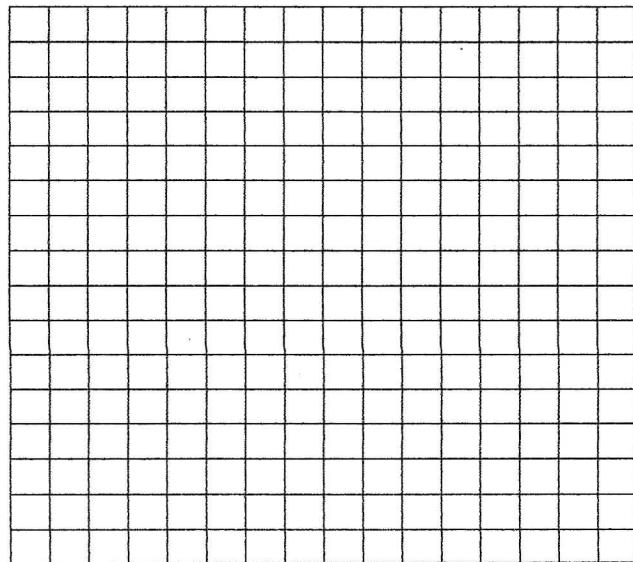
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9) $y = \ln x$ 10) $y = \sin x$ 11) $y = \cos x$ 12) $y = \tan x$ 

13) $x^2 - y^2 = 1$



14) $x^2 + 4y^2 = 4$



XI. SUMMATION

Evaluate each of the following expressions:

1) $\sum_{k=2}^4 (2k - 6)$

2) $\sum_{x=1}^5 |x - 2|$

3) $\frac{2}{3} \sum_{a=1}^4 (a+1)^2$

4) $\sum_{n=0}^{\infty} 3^{1-n}$

XII. SIMPLIFYING FRACTIONS

Simplify each expression:

1)
$$\frac{1+\frac{1}{n}}{\frac{n+1}{n^2}}$$

2)
$$\frac{x-y}{\frac{1}{y}-\frac{1}{x}}$$

3)
$$\frac{\frac{x}{x+1}}{1-\frac{x}{x+1}}$$

4)
$$\frac{\cos x - \frac{1}{\cos x}}{1 - \frac{1}{\cos x}}$$

5)
$$\frac{\frac{a}{2x+h} - \frac{a}{2x}}{h}$$

6)
$$\frac{\frac{\sqrt{x+9} - 3}{x}}{x}$$

7)
$$\frac{\frac{\sqrt{x+1} - 2}{x-3}}{x-3}$$

XIII. DISTANCE FORMULA

1) Find the distance between points $A(-2, \frac{1}{2})$ and $B(1, -\frac{7}{2})$.

2) The distance from point $P(2,3)$ to point $Q(-1,y)$ is 5. Find all possible coordinates for point Q .

3) The distance from point $A(x,3)$ to point $B(10,18)$ is 17. Find all possible values of x .

XIV. INEQUALITIES

Use interval notation to represent the solution set for each of the following inequalities:

1) $x^2 - 2x - 35 \leq 0$

3) $4x^3 - 16x \geq 0$

5) $\frac{(x+1)(x-1)}{x} < 0$

2) $x^2 > 3x$

4) $x^4 - 4x^3 \leq 0$

6) $2x^4 + 3x^3 - 2x^2 \geq 0$

XV. ASYMPTOTES

Find the equations of all vertical ($x = ?$) and horizontal ($y = ?$) asymptotes (if they exist).

1. $y = \frac{x}{x-3}$

2. $y = \frac{x+4}{x^2-1}$

3. $y = \frac{x+4}{x^2+1}$

$$4. \ y = \frac{x^2 - 9}{x^3 + 3x^2 - 18x}$$

$$5. \ y = \frac{2x^3}{x^3 - 1}$$

$$6. \ y = \frac{\sqrt{x}}{2x^2 - 10}$$

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