SUMMER MATH PACKET 1-A

The problems in this packet have been selected to help you to review concepts in preparation for your next math class. Please complete **all problems** in this packet.

- Read the explanations and model problems

- Show work clearly and fully in the space provided

- Hand the packet to your teacher on the first day of class. It will count as a graded assignment

- No calculator for this problem set

- There are 44 problems

Have a great summer and we look forward to meeting you
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Ratio and Proportion

Definition. A *ratio* is an ordered pair of numbers, denoted as either $a : b$ or $\dfrac{a}{b}$ and read as the ratio of a to b.

Generally, ratios relate values in one of three types: *part-to-part, part-to-whole,* or *whole-to-part.*
For example, suppose $a + b = c$, we may write the following ratios as:

part-to-part $a : b$ or $b : a$ In the other notation, $\dfrac{a}{b}$ or $\dfrac{b}{a}$.

part-to-whole $a : c$ or $b : c$ In the other notation, $\dfrac{a}{c}$ or $\dfrac{b}{c}$.

whole-to-part $c : a$ or $c : b$ In the other notation, $\dfrac{c}{a}$ or $\dfrac{c}{b}$.

Examples.

1.In a school with 420 students and 35 teachers:

(a) What is the ratio of students to teachers?   Answer: $\dfrac{students}{teachers} = \dfrac{35}{420} = 35 : 420 : 35 = 1 : 12$

(b) What is the ratio of teachers to students?   Answer: $\dfrac{teachers}{students} = \dfrac{420}{35} = 420 : 35 = 12 : 1$

These are both PART:PART ratios

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2. A school had 180 boys and 240 girls attending.

(a) What is the ratio of students to boys? $420 : 180 = 42 : 18 = 7 : 3 = \dfrac{7}{3}$ This is $\dfrac{whole}{part}$ ratio

(b) What is the ratio of girls to students? $240 : 420 = 24 : 42 = 4 : 7 = \dfrac{4}{7}$ $\dfrac{part}{whole}$

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3.   A recipe calls for 2 parts sugar, 5 parts flour, and 3 parts milk.

(a) What is the ratio of sugar to flour to milk? $2 : 5 : 3$

(b) What is the ratio of sugar to the whole recipe? $2 : 10 = 2 : 10 = 1 : 5 = \dfrac{1}{5}$

Set I

1. Write each of the following ratios in both fraction and ratio form. Simplify as much as possible.

There are 15 apples, and 12 cherries

a) ratio of apples to cherries =

b) ratio of cherries to apples =

c) ratio of cherries to all fruits =

d) The ratio of apples to all fruits =

2) The distribution of final grades in a mathematics class showed 4 A's, 6 B's, 12 C's, 8 D's, and 2 F's. What is the ratio of:

A's to the class =   A's to F's =   C's to A's   the class to C's=

Multiple Choice

Identify the letter of the choice that best completes the statement or answers the question.

3) A platter contains the following items:

Type of Vegetable	Number on Platter
Carrots	10
Peas	11
Peppers	30
Green beans	40

What is the ratio of peppers to peas?

a. 11:30   c. 30:11
b. 91:11   d. 30:91

4. The table shows the number of several types of balls sold by a sporting goods store.

Type of Ball	Number in Store
Baseballs	65
Softballs	3
Footballs	81
Soccer balls	35

What is the ratio of softballs to the total number of balls?

a. 3:65   c. 184:65
b. 184:3   d. 3:184

5. A street contains houses that are painted the following colors:

Color of House	Number of Houses
Red	31
Green	23
Yellow	45
Brown	6

What is the ratio of total number of houses to the number of green houses?

a. 105:23   c. 6:23
b. 105:6   d. 23:105

6. List three equivalent ratios that compare the number of *w*'s below to the number of *g*'s.

       wwwwwwww   *gggggggggggggggggggggggg*

a. $\dfrac{8}{24}, \dfrac{8}{12}, \dfrac{16}{24}$ c. $\dfrac{24}{8}, \dfrac{12}{4}, \dfrac{48}{16}$

b. $\dfrac{4}{3}, \dfrac{16}{48}, \dfrac{8}{48}$ d. $\dfrac{8}{24}, \dfrac{4}{12}, \dfrac{16}{48}$

Proportions: A *proportion* is a statement that gives the equality of two ratios, denoted as either $a : b = c : d$ or $\dfrac{a}{b} = \dfrac{c}{d}$.

    Example.

        The dog to cat ratio for Mathville is two to three. If there are 18 cats, how many dogs are there?

        Solution: We use the ratio $\dfrac{\text{dogs}}{\text{cats}}$ and let *N* represent the number of dogs.

$$\frac{\text{dogs}}{\text{cats}} \qquad \frac{2}{3} = \frac{?}{18}$$

$$\frac{2\cdot 6}{3\cdot 6} = \frac{?}{18}$$

$$\frac{12}{18} = \frac{?}{18}$$

           ?= 12, since the numerators must be equal.
           There are twelve dogs in Mathville.

Cross-Multiplication. $\dfrac{a}{b} = \dfrac{c}{d}$ *if and only if $ad = bc$. This can be used to solve the above problem*

        Let x = number of dogs

$$\frac{x}{18} = \frac{2}{3}$$

           3x = 36
           divide by 3, so x=12

Set II:
Solve each proportion. Show your work.

(7) $\dfrac{3}{8} = \dfrac{n}{56}$

(8) $\dfrac{14}{18} = \dfrac{n}{27}$

(9) $\dfrac{10}{d} = \dfrac{5}{16}$

(10) $\dfrac{9}{c} = \dfrac{3}{53}$

(11) $\dfrac{16}{7} = \dfrac{96}{x}$

(12) $\dfrac{4}{13} = \dfrac{15}{y}$

Set III: Word Problems: Write a proportion for each. Then solve. Show your work!

13. The ratio of boys to girls is 3 to 2. If there are 12 boys, how many girls are there?
    It takes one Super Giant Pizza to feed 3 people. If you invite 36 people, how many pizzas
    will you need?

14. At a recent party, it cost $9.50 for refreshments for 10 guests. At this rate,
    how much would it cost to have refreshments for 80 guests?

15. Mary has saved $17.50 in the past 3 weeks. At this rate, how much will she save in
    15 weeks?

16. Mr. Johnson was paid $190 for a job that required 40 hours of work.
    At this rate, how much should he be paid for a job requiring 60 hours of work?

17. If two pounds of meat will serve 5 people, how many pounds will be needed to serve 13 people?

18. Jack was planting a tree. He was to dig a hole that was 3 feet deep for every 5 feet of tree height. How deep should he dig the hole for a tree that is 17 feet high?

19. A certain shade of green paint is made from 5 parts yellow mixed with three parts blue. If 2 cans of yellow are used, how many cans of blue should be used?

20. If a 4-pound roast takes 150 minutes to cook, how long should a five-pound roast take?

Percents Expressed As Ratios

Percent (%) means per hundred.   (A century is a hundred years - so 20 years is 20% of a century)

Examples : Write percent as a fraction with a denominator of 100 , simplify and write in ratio form

(i) $46\ \% \ \ = \ \dfrac{46}{100} = \dfrac{23}{50} = \ 23\ :\ 50$

(ii) $20\ \% \ \ = \ \dfrac{20}{100} = \dfrac{1}{5} \ = \ 1\ :\ 5$

(iii) $125\ \% \ \ = \ \dfrac{125}{100} = \dfrac{5}{4} \ = \ 5\ :\ 4$

(iv) $1\ \% \ = \ 1/100 \ = \ 1\ :\ 100$

(v) $42.6\% = 42.6/100 = \ 21.3/50 = 21.3 : 50$

We can use proportions to solve percent problems:

i) What is 24% of 50?	iii) If tax is 7.25%, what is the tax on $86
$\dfrac{24}{100} = \dfrac{x}{50}$ $x = 12$	$\dfrac{7.25}{100} = \dfrac{x}{86}$ $100x = (86)(7.25)$ $100x = 623.5$ *divide by* 100 $x = 6.235$ *or approximately* $6.24
ii) 30 is <u>what percent</u> of 150 $\dfrac{x}{100} = \dfrac{30}{150}$ *Can simplify to* $\dfrac{x}{100} = \dfrac{1}{5}$ *so* $5x = 100$ *divide by* 5 $x = 20$	

Set IV: **Answer each of the following by setting up *proportions* and solving:** (see previous page)

15) What is 35% of 200?

16) 18 is what % of 300?

17) There are 100 girls and 150 boys. What percent of all of the students are boys?

18) Tax charged on a new coat is 7.5%. If the coat cost $80, how much is the tax?

19) A car cost $25,000 and the tax paid was $3000. What percent is the tax?

20) Sean took a 45 question test and got 40 questions correct. Approximately what percent did he get correct?

21) John bought a bike priced at $320. The sales tax was 6%. What was the total cost of the bike? (Find the tax and add it to the price of the bike,)

22) 36 students chose to do the extra credit I offered. This figure represents 30% of my students. How many students do I have?

POSITIVE AND NEGATIVE INTEGERS

A. Rules for Adding Positive and Negative Numbers

To add two positive numbers, add and keep the positive sign

      Example: $(+6) + (+7) = +13$

To add two negative numbers, add and keep the negative sign

      Example: $(-13) + (-24) = -37$

To add numbers with different signs, find the difference between the two numbers (subtract) and give the answer the sign of the larger number

      Example #1: $(+17) + (-6) = +11$
      Example #2: $(-32) + (+18) = -14$

B. Rules for Subtracting Positive and Negative Numbers

To subtract signed numbers (either positive or negative), change the subtraction sign to addition and change the sign of the number that follows, then revert back to the addition rules

      Example #1: $(+8) - (+5) = (+8) + (-5) = +3$
      Example #2: $(+7) - (-4) = (+7) + (+4) = +11$
      Example #3: $(-12) - (+6) = (-12) + (-6) = -18$
      Example #4: $(-23) - (-16) = (-23) + (+16) = -7$

C. Rules for Multiplying and Dividing Positive and Negative Numbers

With both multiplication and division, when the signs are the same, the answer will be positive

      Example #1: $(+5) \times (+7) = +35$
      Example #2: $(-5) \times (-7) = +35$

      Example #3: $(+10) \div (+2) = +5$
      Example #4: $(-10) \div (-2) = +5$

When the signs are different in a multiplication or division problem, the answer will be negative

      Example #1: $(+8) \times (-7) = -56$
      Example #2: $(-12) \times (+4) = -48$

      Example #3: $(+9) \div (-3) = -3$
      Example #4: $(-14) \div (+2) = -7$

Order of Operations: PEMDAS

        P- operations inside parentheses
        E- evaluate exponents
      M,D- multiplication and division from left to right
      A,S- addition and subtraction from left to right

Example #1: $-2(12 - 8) + -3^3 + 4 \cdot -6$
               $-2(4) + -3^3 + 4 \cdot -6$
               $-2(4) + -27 + 4 \cdot -6$
               $-8 + -27 + -24$
               $-35 + -24$
               -59

Example #2: $-3 + 4(2 - 6)^2 \div -2$
               $-3 + 4(-4)^2 \div -2$
               $-3 + 4(16) \div -2$
               $-3 + 64 \div -2$
               $-3 + -32$
               -35

If the operations to be performed are in fractional form, solve the numerator first, then the denominator, then reduce.

Example: $\dfrac{7(-4) - (-2)}{8 - (-5)} = \dfrac{(-28) - (-2)}{13} = \dfrac{-26}{13} = -2$

Set V:

23. $18 - (-12 - 3) =$

24. $-19 + (7 + 4)^3 =$

25. $18 + (-7) \cdot (32 - 6) =$

26. $-19 - (-3) + -2(8 + -4) =$

27. $20 + -4(3^2 - 6) =$

28. $-3 + 2(-6 \div 3)^2$

29. $3 \cdot (-4) + (52 + -4 \cdot 2) - (-9.82) =$ 30. $2^3 + (-16) \div 4^2 \cdot 5 - (-3) =$

31. $-6 (12 - 15) + 2^3 =$ 32. $\dfrac{4(-6) + 8 - (-2)}{15 - 7 + 2} =$

Set VI: Solve the following word problems using positive and negative numbers.

33. Steve has overdrawn his checking account by $27. His bank charged him $15 for an overdraft fee. Then he quickly deposited $100. What is his current balance?

34.   Joe played golf with Sam on a special par 3 course.. They played nine holes. The expected number of strokes on each hole was 3. A birdie is 1 below par. An eagle is 2 below par. A bogie is one above par. A double bogie is 2 above par.   On nine holes Frank made par on 1 hole, got 2 birdies, one eagle, four bogies, and one double bogie. How many points above or below par was Franks score?

35.   Find the difference in height between the top of a hill 973 feet high and a crack caused by an earthquake 79 feet below sea level.

36. In Detroit the high temperatures in degrees Fahrenheit for five days in January were -12°, -8°, -3°, 6°, -15°. What was the average temperature for these five days?

37. Hightop Roofing was $3765 in the "red" (owed creditors this amount) at the end of June. At the end of December they were $8765 in the "red." Did they make or lose money between June and December? How much?

38. To establish the location of a hole relative to a fixed zero point, a machinist must make the following calculation:

$y = 5 - (3.750 - 0.500) - 2.375$ Find y.

Substitute and evaluate: $x = -8, \ y = 6, \ m = -3, \ p = \frac{1}{2}, \ n = \frac{3}{4}$

39. $4x - 2m$ 40. $5y + 8p$

41. $nxy \div m$ 42. $2(3x + 6) \div (10m)$

43. $2ny + x$ 44. $(x + y) \div p$